FatLim – new Internet technology for comparing various multiaxial high-cycle fatigue methods

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Abstract

FatLim is an open database accessible to any visitor on http://www.pragtic.com/experiments.php. The database provides experimentally set fatigue limits under multiaxial loading and also the computation results of an equivalent uniaxial load under uniaxial and multiaxial loading at the fatigue limit. At the time of writing, it contains a set of 450 experiments with 18 different computational variants. The database is currently equipped with a simple selection feature, enabling the user to refine the search for a group of tests of a similar nature. Use of the database system is absolutely free, without any fees. The weak points of the solution presented here, as regards both the system and its contents, are described and discussed.

Keywords: multiaxial fatigue, high-cycle fatigue, mean stress effect, non-proportional loading

Nomenclature

C	[MPa]	shear stress on an examined plane
ΔFI	[%]	fatigue index error (see Eq. (2))
f_{-1}	[MPa]	fatigue limit in fully reversed axial loading
t_{-1}	[MPa]	fatigue limit in fully reversed torsion
N	[MPa]	normal stress on an examined plane

1. Introduction

A wide range of methods have been proposed for comparing local constant amplitude loads with a fatigue limit under simple uniaxial loading. Although the refining process has already taken more than 50 years, it does not seem to have reached a satisfactory conclusion, and new criteria are still being proposed every year. It is therefore appropriate to ask why there are so many methods, and whether there is not one that can be recommended as the best for some specific situation or under some specific conditions, so that newly introduced methods can use it as a benchmark.

Current approaches suffer from the following issues:

1. **Tradition:** Although so many methods of various ages belong to this category, substantial comparisons with newer methods are lacking. New methods are usually compared with, and compete with, only the simplest, oldest or weakest methods (typically the criteria by Papadopoulos [1], McDiarmid [2], Sines [3], Crossland [4] or Matake [5]), probably due to the high demands on the complexity of multiaxial solutions, where an easy analytical solution is only rarely available.
2. **Narrow scope:** The number of experimental data points used for the evaluation tends to be very small – usually only 40-50 experiments ([1], [6], [7] or [8]). If the testing set is larger, there is no detailed description of the experiments ([9], which claims to make use of 179 experiments).
3. **Lack of rigour:** The experimental data used for the evaluation is often of a type that enables most methods to behave nicely. Typical representatives of such mild experimental data are those published by Nishihara and Kawamoto [10] or by Gough (excessively cited among an otherwise appealing number of 108 experiments in [11]). Experiments where the prediction results are less

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easily correlated with measurements are not transmitted to other papers, or the data is even removed from the experimental data set (some of Findley's experiments from [20] as referred to in [11]).

4. **Carelessness:** There are too many errors induced by careless reproduction of the experimental data (e.g. Ninic in [11] systematically confuses British and US tons). The damage accumulation law is as valid here as it is in fatigue assessment – any new error induced by careless transcription is reproduced further to other papers. Under given conditions, it is essential to check with the original data source.

If research in the area of fatigue computation is to progress, new criteria should be subjected to increased demands. In order to enhance the quality of new criteria, the critical experimental results should be easily available and, if possible, free of error.

2. The concept of FatLim

Use of the PragTic[1] freeware, developed by the author, is a way to deal with point 1, above, as it enables rapid implementation of new criteria into its existing highly sophisticated structure. Careful examination and long-term collection of the original papers describing experimental results covers the other points mentioned above. Such a complex solution can work in the short and medium term, but it is not sustainable over a longer period of research. It lacks continuity, because it depends on individual researchers. Any closed personal database can be given to other researchers, but this can lead to a set of non-compatible databases of experimental results developed in various parts of the world. In addition, it is hard to reproduce data on such a scale – the only solution is some kind of handbook. This, in turn, can lead to a dead end, if no further changes and modifications are allowed.

A freely accessible Internet database seems to be an appealing solution. Several important considerations have to be taken into account:

1. Make it accessible to any person on this planet that is connected to the Internet.
2. Enable new data to be inputted, in order to enhance continuity.
3. Reserve such input rights for selected persons (a committee), or award them to anybody who volunteers.
4. Ensure that the data items gathered in the database are correct, and that there are no mystifications.

3. FatLim implementation

FatLim is an acronym standing for the Database of Fatigue Limits. It and all the other databases adjoined to it are available on the top right corner of the http://www.pragtic.com website.

3.1. Scope

At the time of writing, FatLim covers experimental results from 450 experiments. These are either uniaxial experiments with non-zero mean stress, or multiaxial experiments. It was originally assumed that the collected experiments had been performed on smooth unnotched specimens, in order to exclude the effect of the non-homogeneous stress field around the notch, and the effects of different surface treatments. Until further expansion to the area of notched components occurs, this limitation should be observed[2].

The loads are described by the amplitude and mean stress on each load channel (two axial, one in torsion), their phase and period. As harmonic loading is the prevalent loading in such experiments, this is the only option, except for the static loading that forms the mean stress. No limitation is imposed on the material, but the definition assumes that the material is isotropic.

The 18 computational methods used for the calculation itself are described in Help for PragTic[3]. New methods cannot be added from the interface, but have to be inserted by the website administrator.

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[1] PragTic is a freeware for fatigue damage computation, which can also work on FEM-data. It can be downloaded from http://www.pragtic.com/program.php.

[2] As the data was originally included from various available sources, the data provided by Simbürger [12] for notched specimens loaded in bending and torsion was also included, and this fact was discovered only later. The scatter of the results of various methods is not very high for this data set, and the results were therefore allowed to remain, although they fall outside the original scope of the project.

[3] Help for PragTic software can be accessed through the interface on http://www.pragtic.com/program.php, where it can be either directly viewed or downloaded.

3.2. Structure

The basic structure that has to be referred to is **References** (see Fig. 1). It is necessary to keep the database clean from erroneous data of various types, to specify the origin of the data, and to avoid offending against copyrights. All items concerning experimental or material data must therefore refer to the original paper in which it was published. The interface of the reference system allows papers in journals or proceedings, of books, theses to be inputted and also some basic searches.

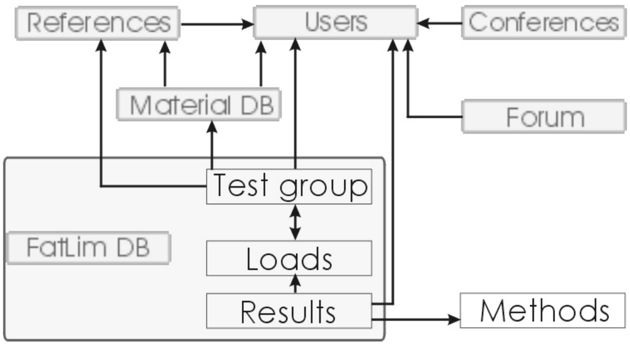

Fig. 1. A set of the most important databases on www.pragtic.com and their mutual dependencies. The databases with rounded corners are directly accessible from the website interface. The databases with sharp corners are hidden – they are either set hard or use a common user interface (FatLim DB here).

The second level is formed by the database of material parameters (**Material DB**). Here the basic static and fatigue characteristics of individual materials are gathered, as reported in various papers, accompanied by a description of the chemical composition and references to the papers in which they were found. A problem can occur here, because some of the material properties for a particular material may be described in a series of papers, unfortunately sometimes also stating different values for a material that nevertheless appears to be of the same origin and charge. For this reason, three references can be set for a single material data set and a further commentary can be inputted.

Once the material properties for the set of experiments are defined, its own definition can proceed (**FatLim DB**). Each group of experiments is once again related to a referred paper, and also to a specific material already defined in the database. Only when the individual experiments in the group have been defined can the computational results for each criterion be described on the specified experimental point.

3.3. The decisive variable

Most of the methods defined in this category can be described as an inequality, where the left hand side contains the load parameters – most often a combination of shear and normal stresses on a plane. As mentioned in [13], the unique material parameter should be placed on the right hand side in order to allow a comparison with other criteria. The criteria can be transformed to such a form that the right hand side contains only one material variable – the fatigue limit in fully reversed axial loading:

$$L.H.S.(load) = a \cdot f(C) + b \cdot g(N) \le f_{-1} = R.H.S.(material) \ . \qquad (1)$$

The material parameters *a* and *b* given here, or some other parameters, relate the weights of the damage effects of the shear *C* and normal *N* stresses.

The author intentionally writes a general fully reversed axial loading. In order to minimize the number of interacting effects in the evaluation, the type of axial loading under uniaxial testing (run for retrieval of the material parameter) and under multiaxial testing should coincide. Ideally, when evaluating the case of bending, the same condition should also be valid for the diameters of the specimens.

An ideal value of unity should be obtained for all tests collected in the FatLim database, if the experimental multiaxial loading is equivalent to the fatigue limit. This is usually not the case – neither the calculations nor the experiments are ideal – and thus the fatigue index error is defined as:

$$\Delta FI = \left(\frac{L.H.S.(load) - R.H.S.(material)}{R.H.S.(material)} \right) \cdot 100\% \ . \qquad (2)$$

Because the load influence is placed on the left hand side, fatigue index error values below zero denote that the criterion does not predict fracture, although it actually occurred during the experiment. Such a prediction has to be ranked as non-conservative. Values that are higher than zero correspond to conservative predictions. The fatigue index error defined in Eq. (2) can be interpreted as the relative deviation of an equivalent load at the multiaxial fatigue limit from the uniaxial fatigue limit in fully reversed axial loading.

The prediction results stated in FatLim are all obtained from the computation realized in PragTic freeware. The tool used for the computation has to be noted when the results are inputted into FatLim. There is also an option to use, e.g., an analytical formula. The use of any other computational system has to be announced to the administrator of the website, so that he can add it to the available computational tools.

3.4. Safety measures –position of the manager, editing, signing

To boost up the further growth of the databases, it is desirable to allow any volunteer to input new values. However, there needs to be some safety policy that ensures that the data inputted into the system is correct and that there are no mistakes, either intentional or casual.

The first rule in the system is that only so-called privileged users are allowed to change the content of the databases. Privileged users differ from anonymous users by the fact that they agree to a brief description of their background in the **Users** section. This is not of course a sure solution, because it is no problem to fake an Internet identity today.

If somebody inputs a new data item into the system, she/he automatically accepts the function of its **manager**. From now on, she/he is the only person (except for the administrator) that is allowed to directly modify the data, and her/his username is clearly attached to all her/his items. Other users can mark the validity of the input data by **signing** it, thus indicating their approval through specific records: stating that they have seen the original paper and compared it to the Internet representation, and that everything coincides. Logically, the number of signs is directly related to the level of verity.

This is the current state, but it is not the final version. If any user finds data that is evidently wrong, she/he must have the opportunity to report doubts to other users, or to propose a different version. A final state of FatLim is envisaged in which the user can record a counter-proposal that will be saved to the system and will remain available to any user who visits the controversial item. The manager of the disputable item will be notified about the counter-proposal by e-mail, and will have a limited period of time (30 days) to either accept it or reject it. If he rejects it, the proposer can either repeat the process or take the case to the **Forum** section, in order to post the problem to other users. If the manager takes no action, the managerial right is passed on to the proposer and her/his proposal is accepted. This rule is important, as it makes it possible to adopt items abandoned by users not longer active in the FatLim system.

3.5. Handling the databases

Any new data can be inputted into the system only by a registered person, which is currently logged in. After checking that they are not already there, the first step is to input the references to all the original papers from which the data is excerpted through the interface on the **References** website. If the references relate to a paper in a journal or in proceedings, the journal or proceedings should already have been defined. If this has not already been done, these references must be inputted before the paper can be inputted.

Once the references are defined, the user can go on to define the material parameters referred to in the paper on the **Material DB** webpage. The database is built on the assumption that only data directly reported in the referred papers is inputted. No further post-processing or assembling of a complete material description from other papers is accepted. Such compilation is the responsibility of users of the databases.

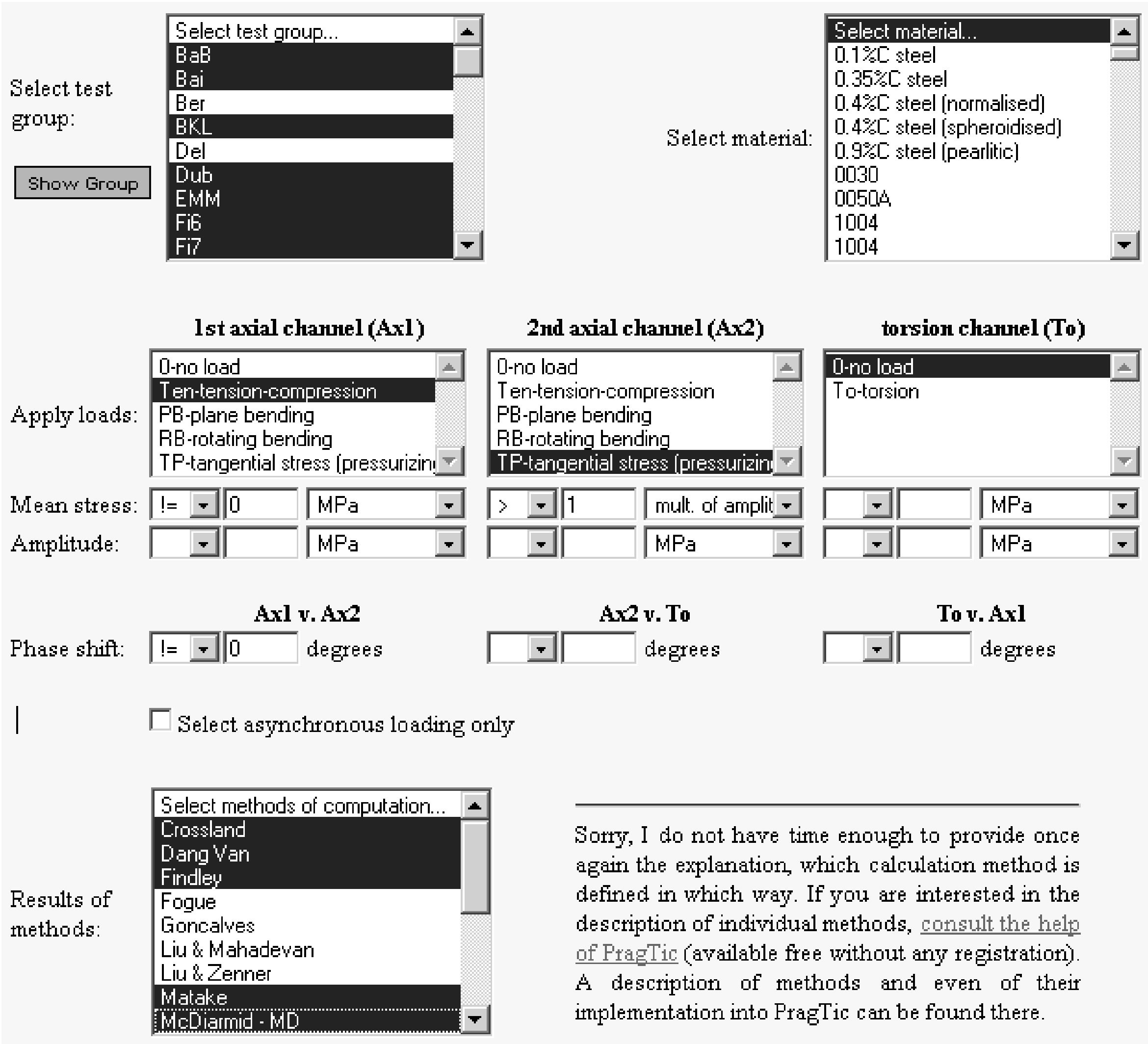

Fig. 2. A screen copy of the FatLim DB web page with the dedicated selection interface.

The next step is to define a group of experiments on the **FatLim DB** webpage. This is done in two phases. First, the name of the test group is set. A combination of several characters derived from the names of the authors is preferred, to enable quick orientation among the various experimental sets. The material and the appropriate paper are referred to, and the number of individual experiments in the particular data set is given. A definition of the individual items proceeds after confirmation of the data group properties that have already been set.

This is the end of the definition of the experimental data set. To complete the FatLim content, the impact of the individual methods on the experimental results also has to be inputted. This is also done from the interface of the **FatLim DB** website. The acronym of the test group is selected, and the adjacent **Show Group** button is pressed. This command opens a new page (see Fig. 3) with a detailed description of the selected test group, where the prediction results can be opened for editing by clicking on the **E** button in the row corresponding to the chosen calculation method. The group of experiments and the calculation results can be signed from this interface.

Bai	Manager: jan	To do: E RS	
Referenced in:	Baier, F.: Zeit- und Dauerfestigkeit bei ueberlagerter statischer und schwingender Zug-Druck- und Torsionbeanspruchung. [PhD thesis]. Stuttgart, Universitaet Stuttgart 1970. **Manager:** jan		
Material:	34CrMo4 (ID=12)		
Comments:	Note please that the calculations were done for a section through the available S-N curves derived from Baier's graphs (he does not state anywhere precise numbers). The section was done at N=8e5, where the most of the S-N curves were clearly defined by experiments. The respective fatigue endurances/limits used for the calculations thus are: f-1=398MPa, f0=646MPa, t-1=295MPa, t0=518MPa. Due to the way of data derivation, some error in their retrieval has to be expected, but the overall statistics and behavior under extra high or low mean stresses is clear.		
Signed by:	jan		
Note: E ~ edit; S ~ sign; RS ~ remove sign			
Method	**Manager**	**To do**	**Signed by**
Crossland	jan	E RS	jan
Dang Van	jan	E RS	jan
Findley	jan	E RS	jan

Fig. 3. Description of the test group with experiments provided by Baier [14]. Note the E, S and RS buttons used for editing, signing or sign removal.

4. Discussion

4.1. Technology

This paper does not aim to analyze the results available from the statistical data gathered in the FatLim database. Not all data is currently adopted from primary sources – some has been read from papers that only quote from the original, or even from another source. In addition, not all the computational results are available. Finally, the author is until now the only manager active in the database, and only his signatures can be found everywhere. If the system is to work properly, considerable participation by other users is desirable.

This paper refers above all to the technology for distributed sharing of experimental research data. The methodology described here aims to deal with the weak points mentioned at the beginning of the paper. The user can quickly understand the limits of existing methods, and the system provides a good starting point for any new study, e.g. by a PhD student. Another important consideration for new researchers is that the necessary information is provided free of charge. Finally, the information is relatively clearly structured, and is also accessible for people working in industry. They can quickly understand whether the methods that they have been using, e.g. in commercial fatigue post-processors, can provide satisfactory results, or whether they should change their current practices.

An apparent weak point of the current system is related to its free availability. To keep the system intact, it is necessary to work on maintaining and developing it. However, this is not a critical problem, as the costs are not very high. More important is the need for volunteers to replenish the content of the databases. Only in this way can the original purpose of the database – data sharing – be achieved.

4.2. Fatigue limit

The complete evaluation of a fatigue limit is a time-consuming, demanding and expensive task. For this reason, only a part (probably a small part) of the data gathered in FatLim consists of real fatigue limits, as evaluated by the staircase method.

Other solutions are more often used. Several points on the inclined slope of the S-N curve are set experimentally and the number of cycles corresponding to the fatigue limit is set hard. This solution can lead to problems, since the number of cycles at the fatigue limits for push-pull and for torsion can differ (see e.g. [14]). It is then debatable whether the mechanisms leading to the damage, which in this case are entirely different for the two types of loadings, can be used for the final assembly of the equivalent uniaxial load. Under high mean axial loads, moreover, the fatigue limits shift considerably to a much lower number of cycles [14]. In some other papers, the authors focus on the inclined part of the S-N curve only, and do not deal with the particular value of the fatigue limit.

So far, there has been no discussion about the way of retrieving the fatigue limits for all the data reprinted so often in various research papers. However, this is essential, in order to ensure the validity of FatLim. The final solution accepted in FatLim in such cases is that a section is led through the appropriate S-N curves, ensuring that the basic fatigue curves (fully reversed axial loading, fully reversed torsion loading, repeated axial loading) are all evaluated in the inclined part not far from the fatigue limit.

There are also various experiments with multiaxial loading at different frequencies (see e.g. [15]). Such experimental points are also covered in FatLim, and the selector (Fig. 2) directly enables these experiments to be separated from all the other experiments. However, their inclusion in FatLim is controversial. The complex load path defined by more than one load channel acting with different frequencies is likely to be segmented to more than one load cycle. Damage summation cannot work here, because only one variable of the S-N curve – the fatigue limit – is set. The authors of [15] are aware of this problem, and compare the results of such tests only in relation to the basic experiments with an iso-frequency loading.

Several problems highlighted in the discussion need to be resolved. The criteria should not be evaluated at the fatigue limit but with the S-N curve within the high-cycle fatigue region, where the plasticity effect is not so pronounced. However, this introduces problems with the selection of a proper damage accumulation law, and with the behavior of the S-N curve below the fatigue limit level.

4.3. Completeness

Although the individual items of FatLim and also the overall statistics on all the prediction results are not discussed here, it is necessary to emphasise the limitations of the current content as regards complete coverage of various materials and load effects. Only harmonic or static loading is allowed on any load channel. Even if the interface and the database structure will be developed further, the author has found little data with fatigue limits obtained with another type of load course.

A problem of higher priority is the representation of various materials. The only condition enforced by the FatLim structure is that the material is isotropic. But when the observer goes through the content, a startling fact is that most of the materials described are steels with a ratio of fatigue limits in fully reversed axial loading f_{-1} to fully reversed torsion t_{-1} close to $f_{-1}/t_{-1} = 1.6$. There are almost no extra-ductile steels. The number of tests on cast irons is higher, but a more thorough look reveals that there are only a few tests with real non-proportional loads, and that there is also no evaluation of the effect of higher mean stresses. The aluminum alloys that are used increasingly massively nowadays end up with the same score. While the present state of FatLim can help us to understand the behavior of various fatigue computational criteria on steels, the actual response for non-ferrous alloys as well cast irons can be only estimated.

4.4. Data sets for benchmark

The FatLim database is not complete as regards checking with individual original data sources, and also the computational results of some methods. These two points need to be finalized, in order to define a benchmark with a lesser scope. Such a group of experiments is an appropriate solution for a simple test of the newly-proposed calculation methods. In this way, their authors will not be overwhelmed by the work that would be needed for a functionality proof of their solution. Before the FatLim check and completion are finished some comments on the test groups recommended for similar tests are presented:

1. The mean stress effect is much more pronounced than the effect of non-proportional loading under the condition of zero mean stress if the quality of the prediction of various computational methods is evaluated.
2. The data provided by Nishihara and Kawamoto [10] (NKc, NKd, NKh, NKm groups in FatLim) covers only the effect of non-proportional loading, and no mean stresses are applied. Its use in any tests of new computational methods has no predicative value.
3. The extensive testing carried out by Gough and Pollard [16], [17] (GPA-GPR in FatLim) on the basic response of various cast and structural steels has a similar disadvantage, which is even more pronounced due to lack of non-proportional loads. Only the more extensive evaluation of S65A steel reported by Gough in [18] (Ggh in FatLim) with non-zero mean stresses has an appropriate predicative value.
4. There are very high mean stresses in the experiments reported in the paper by Bomas et al. [19] (BKL in FatLim). The related experiments follow a different trend from the data with a similar magnitude of mean stresses referred by Baier [14] or Findley [20], and significantly worsen the prediction results of some methods. Further experiments with a similar setup are desirable.
5. The test groups by Findley [20] (Fi6, Fi7 and Fi8 in FatLim) must also include the cases of high mean stresses. Their removal makes the test group worthless as regards the search for the best solution.

6. The data sets provided by Baier [14] (test groups Bai and BaB in FatLim), by the research group around Heidenreich (test groups HeG [21], Hei [22], HRZ [23], HZ [24] in FatLim), by Simbürger [12] (SiB in FatLim as regards the unnotched specimens), and by Troost et al. [25] (TAK in FatLim) include various combinations of all interacting effects. This data was checked with the original papers and is recommended for similar analyses.

The reasons for the notes mentioned previously can be readily found by anybody who checks the FatLim content and the resulting statistics.

5. Conclusion

A new database system for referring experimental results and computational predictions in the domain of multiaxial high-cycle fatigue has been introduced. It is intended to serve as a basis for comparing the wide range of approaches used for defining a uniaxial load equivalent to the multiaxial loading at the fatigue limit. The complex solution presented here should:

- gather together most of the experimental data available in this category using the joint efforts of various research institutions;
- converge to a state where no errors are induced in the records of the individual items, thanks to the editing policy;
- provide a starting point for any new research project in this area (e.g., a PhD project);
- optimize the development of new methods by removing the necessity to test the older criteria at every research institution.
- tighten up the demands on new criteria as regards their proof of exceptionality;
- inform engineers from industry on the limits of the methods currently implemented in commercial software;

The management policy used by FatLim allows any volunteer to input new items. The lack of people currently contributing to the system is the biggest problem of the system at the present time.

Two problems are of the highest importance as regards the state of the art concerning coverage of the high-cycle fatigue area. Firstly, the relation of the criteria examined here to real fatigue limits. A better solution for future research could be to work with the S-N curve, and not only with the fatigue limit value. Secondly, there is very unequal representation of various types of structural materials. Currently, steel is the only material that is described adequately. The search for data on other materials (cast irons, aluminium alloys, titanium alloys, etc.) needs to continue, and researchers and industrial engineers should keep this in mind.

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